Determination of Thermal Conductance of Dielectric-Filled Strip Transmission Line from Characteristic Impedance

***Abstract*—The thermal conductance and electrical capacitance of nonmagnetic dielectric-filled TEM transmission lines are related to very close approximation by the simple formula**

$$G/l(\mathrm{W/°C\cdot cm}) = \frac{377\cdot g_{\mathrm{therm}}(\mathrm{W/°C\cdot cm})}{Z_0(\mathrm{ohms})\sqrt{\varepsilon_r}}.$$

Where ε_r is the relative dielectric constant of the dielectric material and g_{therm} is its thermal conductivity. Hence, the thermal conductance of a section of line is the product of the length of the section of line l, in centimeters and G/l, the conductance per unit length.

Several applications of this useful formula are given.

The calculation of the thermal conductance of a thermal conducting path between two bodies and the electric capacitance or conductance between these same bodies reduces to essentially the same mathematical problem.[1] Fig. 1 shows a structure to help visualize the measurement of electrical capacitance and thermal conductance for a block of material with (possibly metal) plates of Area A on two faces, forming thermally and electrically conducting boundaries. The necessary analytical steps carried out for this very simple rectangular geometry show that the electrical capacitance is

$$C_e = \epsilon\frac{A}{w}$$

where ϵ is the absolute dielectric constant and w is the sample thickness. Similarly, the thermal conductance

$$G = g\frac{A}{w},$$

where g is the thermal conductivity.

From these simple examples, we can easily derive the capacitance from the thermal conductance, or vice versa, since the purely geometric factors common to both expressions disappear from the ratio

$$\frac{G}{C_e} = \frac{g}{\epsilon}.$$

This relationship is predicated on the assumption that the metal plate is a good thermal conductor in relation to the dielectric, so that the thermal field pattern is indeed analogous to the electric field pattern. With some high thermal conductivity dielectrics, such as Beryllia (BeO), this question must be examined quantitatively and will perhaps call for a numerical modification of the results.

Because of the general analogy between the solutions to heat flow, current flow, and electrostatic field problems by means of potential methods and Laplace's equation, this relationship can be extended to homogeneous dielectric cross sections of arbitrary shape. The formal derivation of this has been omitted for brevity, but a conformal mapping of a rectangle into an arbitrary outline could always be assumed, and the solution would follow by means of counting series and parallel paths of "curvilinear squares" as discussed below.

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[1] This is discussed in various texts and handbooks, e.g., G. J. Murphy, D. J. Shippy, and H. L. Luo *Engineering Analogies.* Ames, Iowa: Iowa State University Press, 1963.

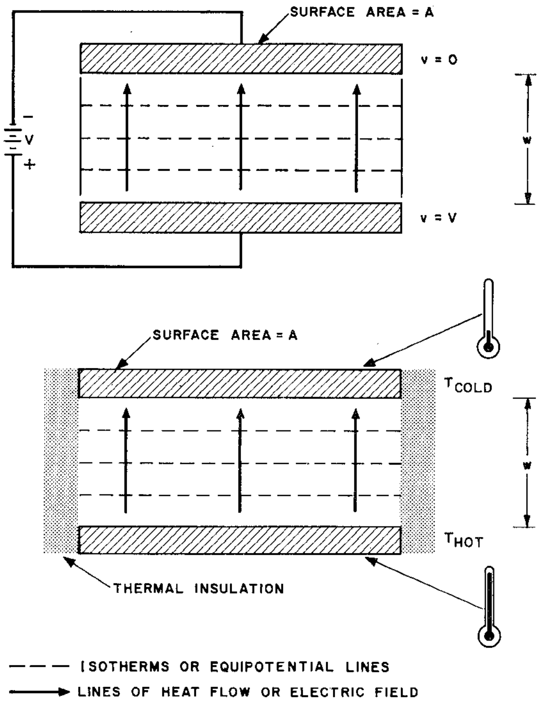

Fig. 1. Simple analogous electrical capacitor and solid block heat flow.

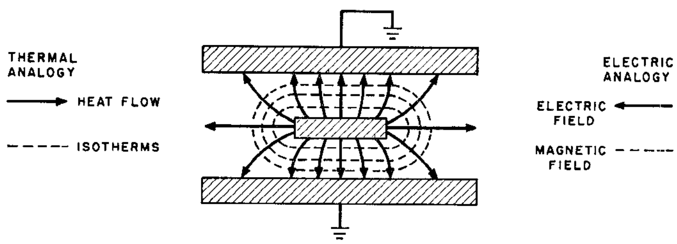

Fig. 2. Schematic view of TEM field in a stripline or of the thermal flow pattern by analogy.

There only remains to relate C_e to Z_0, the characteristic impedance of a TEM transmission line. The primary constants of the TEM transmission line are L (inductance per unit length), and C (capacitance per unit length). These fundamental constants enter into two important formulas, determining the wave speed

$$s = \frac{1}{\sqrt{L\cdot C}}$$

and the characteristic impedance

$$Z_0 = \sqrt{\frac{L}{C}}.$$

By direct substitution,

$$C = \frac{1}{Z_0\cdot s}.$$

Since most dielectrics are nonmagnetic, $s = s_0/\sqrt{\epsilon_r}$ where s_0 is the speed of light in vacuum and ϵ_r is the relative dielectric constant, leading to

$$C = \frac{\sqrt{\epsilon_r}}{Z_0 s_0}.$$

Then

$$G/l = \text{thermal conductance/unit length}$$
$$= \frac{g\sqrt{\epsilon_r}}{\epsilon Z_0 s_0}.$$

This relation can take other, possibly more useful, forms. Since $\epsilon = \epsilon_r\epsilon_0$ and $s_0 = 1/\sqrt{\mu_0\epsilon_0}$ where μ_0 is the magnetic permeability and ϵ_0 the absolute dielectric permittivity of vacuum,

$$G/l = \frac{g}{\sqrt{\epsilon_r}\,Z_0}\sqrt{\frac{\mu_0}{\epsilon_0}}.$$

As is well known, free-space characteristic impedance is $\sqrt{\mu_0/\epsilon_0} \approx 377$ ohms, of this equation can be given as

$$G/l = \frac{g\cdot 377}{\sqrt{\epsilon_r}\,Z_0}$$

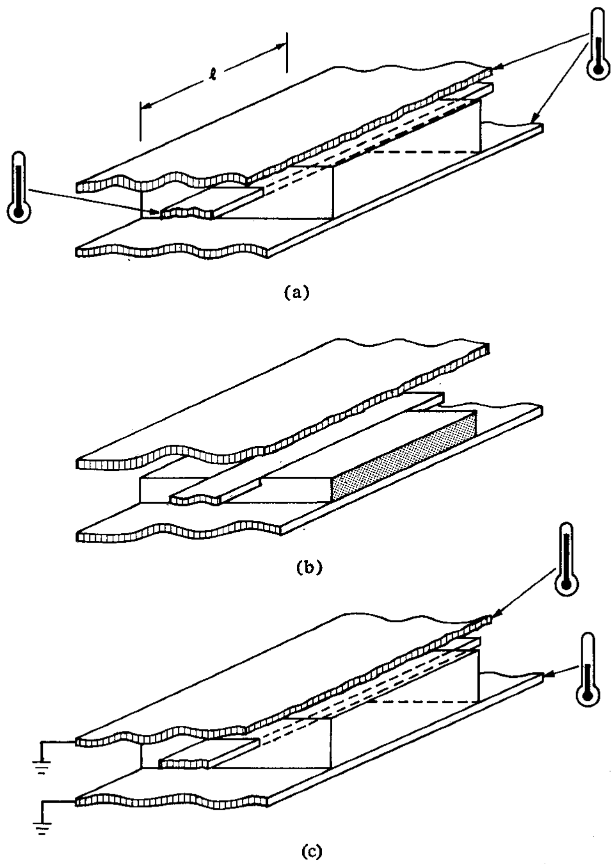

Fig. 3. Valid and invalid applications of the formula. (a) Valid. (b) Not valid—nonhomogeneous dielectric. (c) Not valid—equipotential not analogous to isotherm.

TABLE I

CHARACTERISTIC IMPEDANCE OF SEVERAL COMMONLY USED TRANSMISSION STRUCTURES

Circular coaxial cable

$$Z_0 = \frac{138}{\sqrt{\epsilon_r}} \log_{10}\left(\frac{b}{a}\right)$$

where

a = inner radius
b = outer radius
ϵ_r = relative dielectric constant.

Parallel sheets

$$Z_0 = \frac{377}{\sqrt{\epsilon_r}} \frac{d}{w}$$

where

d = plate separation
w = plate width (valid for $w \gg d$).

Parallel circular wires (equal diameters)

$$Z_0 = \frac{120}{\sqrt{\epsilon_r}} \cosh^{-1} \frac{s}{d}$$

s = spacing between centers
d = wire diameter-

Circular wire centered between ground planes

$$Z_0 = \frac{138}{\sqrt{\epsilon_r}} \log_{10} \frac{4h}{\pi d}$$

where

d = wire diameter
h = plate separation (valid for $d < 3h/4$).

For additional cases see Matthaei, Young and Jones, *Microwave Filters*. New York: McGraw-Hill. *Microwave Engineer's Handbook*. New York: Horizon House.

This formulation can be visualized with the aid of Fig. 2. The electric and magnetic field lines are analogous to the heat flow lines and isotherms, respectively. These two sets of lines divide the transmission-line cross section into "curvilinear squares." These squares have an impedance equivalent to a sheet resistance of $(377/\sqrt{\epsilon_r})$ ohms per square, and the series and parallel connection of these squares makes up the impedance Z_0. Thus, the factor $377/\sqrt{\epsilon_r}Z_0$ is the same geometrical factor we would find by such methods as resistive sheet analogs, after correction for the specific sheet resistance.

Several important facts should be noted about the formula just derived. First, the actual thermal comductance of a sample, such as shown in Fig. 3(a), is

$$(G/l)\cdot l = G\left(\frac{\text{W}}{°\text{C}}\right).$$

where l is the length of the solid material.

Second, the Z_0 in the formula must be the Z_0 *with the dielectric in place*, related to the Z_0 for an *air* line by

$$Z_{0_{\text{dielectric}}} = Z_{0_{\text{air}}}/\sqrt{\epsilon_r}.$$

The derived formula can be restated as

$$G/l = \frac{g\cdot 377}{Z_{0_{\text{air}}}}$$

Third, this formula computes only the heat transfer due to conductance, *not* that due to radiation and convection. Because the analogous equations must be linear, with respect to the independent variable, the dielectric material must have a linear dielectric field-displacement relationship, and the thermal flux-temperature relationship must also be reasonably linear. Furthermore, we assume that the material is isotropic, homogeneous (not partly air and partly solid nor composed of two different dielectrics[2]), and that all points on the equipotential electrical conducting planes are at equal temperature. See Fig. 3.

APPLICATIONS

Any 50 ohm stripline[3] with 99.5 percent (Al_2O_3) dielectric, with $g = 0.27$ W/°C·cm and $\epsilon_r = 9.5$, will have a value[4]

$$G/l = \frac{0.27 \times 377}{\sqrt{9.5} \times 50} = \frac{102}{154} = 0.66 \text{ W/°C}\cdot\text{cm}$$

independent of shape. The only way to get an improved thermal resistance is to use a material with a larger g and smaller ϵ_r, or a line of smaller Z_0.

For example, for BeO of 95.5 percent density, $g = 2.5$ W/°C·cm, $\epsilon_r = 6.6$, so

$$G/l = \frac{2.5 \times 377}{\sqrt{6.6} \times 50} = \frac{945}{128} = 7.35 \text{ W/°C}\cdot\text{cm}$$

while for a low density 85 percent Al_2O_3 with $g = 0.041$ W/°C·cm and $\epsilon_r = \sqrt{8.2}$

$$G/l = \frac{0.041 \times 377}{\sqrt{8.2} \times 50} = \frac{15.4}{143} = 0.108 \text{ W/°C}\cdot\text{cm}$$

CONCLUSION

The thermal conductance of a dielectric-filled stripline can be found more easily by this formula than by previous methods, since it relates the electrical and thermal properties directly, without considering geometry. This development will be found useful in connection with heat transfer studies for stripline systems. The thermal conductance is determined completely by the Z_0, ϵ_r, g, and length l of the dielectric. It follows that any methods proposed for changing the thermal properties, without changing these determining factors, are demonstrated to be ineffective.

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[2] Unless their ϵ_rs are in the same ratio as their gs.
[3] This correspondence refers, throughout, to $Z_{0\,\text{dielectric}} = 50$ ohms.
[4] Data Sheet 0001, Coors Porcelain Co., Golden, Colo.